EXERCISE 1

- Write a Matlab function `[`$\mathbf{v}_k$`,`μ_k`]=power(A,maxit,tol)` that implements the power method to approximate the dominant eigenvalue μ_k of the matrix `A` and related eigenvector $\mathbf{v}_k$. Use a random vector as initial guess and test it on the matrix

  ```
  A = Q*diag(1:10)*inv(Q), with Q = orth(randn(10,10))
  ```

  Note that the spectrum of `A` is $\{1, \dots, 10\}$. Modify the function to be able to plot the relative error $|\lambda_1 - \mu_k| \,/\, |\lambda_1|$ and also $(|\lambda_2/\lambda_1|)^k$, $k = 1, 2 \dots$, and comment.
- Same as above, but test on the nonsymmetric matrix obtained using `Q = randn(10,10)`. Compare the results with the previous case.
- Write the function `inverse_power` that implements the inverse power method (use "backslash" to solve the linear system), and test it on the matrix of the previous point to approximate $\lambda_9 = 2$ using the shift $\mu = 1.55, 1.65, 1.75, 1.85, 1.95$.
  Plot the error $|\lambda_9 - \mu_k|$ vs k for the different shift μ.
  Optional exercise: plot the number of iterations required to converge vs. the value $|\lambda_9 - \mu| \,/\, |\lambda_{10} - \mu|$ and comment.